Greenway Primary & Nursery School

GREENWAY PRIMARY AND NURSERY SCHOOL

WRITTEN CALCULATIONS POLICY

Aims

- To outline for teaching staff and parents the written strategies for calculation taught at Greenway Primary School for addition, subtraction, multiplication and division, in line with the new curriculum for Mathematics.
- To show how using key pieces of practical maths apparatus helps to accelerate the children's learning.
- To ensure consistency of approach from one year group to the next.
- To enable children to develop confidence and fluency in calculations that they will be able to apply to a variety of problem-solving activities.

The importance of mental mathematics

While this policy focuses on written calculations, taught from Year 1, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. These are taught in a wide variety of ways from the Foundation Stage onwards.

Progression in Addition and Subtraction

Addition and subtraction are connected.

Part	Part
Whole	

Addition names the whole in terms of the parts and **subtraction** names a missing part of the whole.

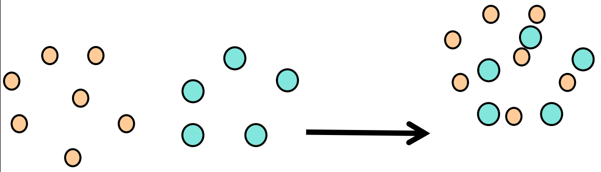
ADDITION

From Year One

Combining two sets (aggregation)

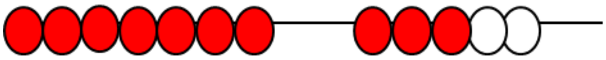
Putting together – two or more amounts or numbers are put together to make a total

7 + 5 = 12



Count one set, then the other set. Combine the sets and count again. Starting at 1.

Counting along the bead bar, count out the 2 sets, then draw them together, count again. Starting at 1.



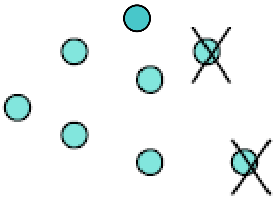
SUBTRACTION

From Year One

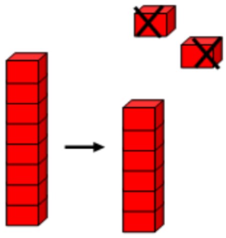
Taking away (separation model)

Where one quantity is taken away from another to calculate what is left.

8 – 2 = 6



Multilink towers- to physically take away objects.



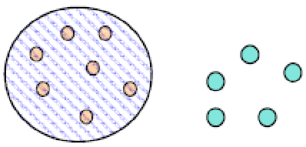
ADDITION

Combining two sets (augmentation)

This stage is essential in starting children to calculate rather than counting

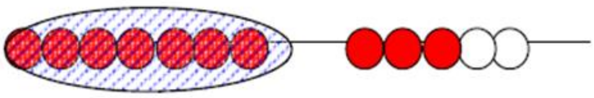
Where one quantity is increased by some amount. Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number.

Counters:



Start with 7, then count on 8, 9, 10, 11, 12

Bead strings:



Make a set of 7 and a set of 5. Then count on from 7.

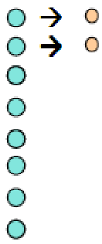
SUBTRACTION

Finding the difference (comparison model)

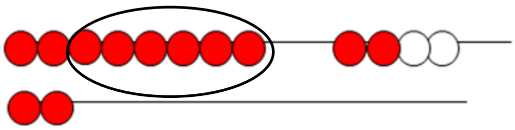
Two quantities are compared to find the difference.

8 – 2 = 6

Counters:



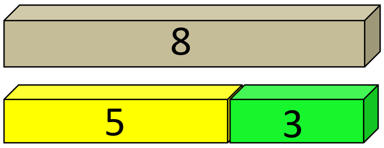
Bead strings:



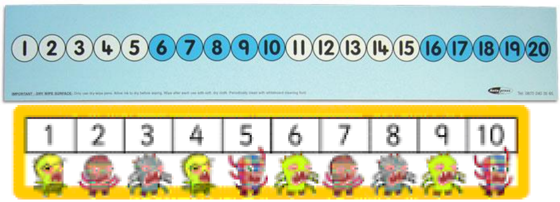
Make a set of 8 and a set of 2. Then count the gap.

Multi-link Towers:

Cuisenaire Rods:

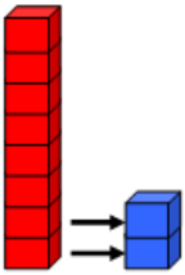


Number tracks:

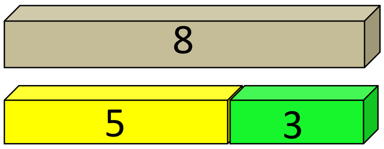


Start on 5 then count on 3 more

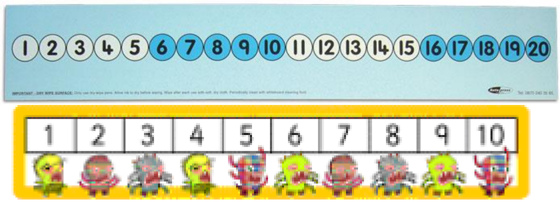
Multi-link Towers:



Cuisenaire Rods:



Number tracks:



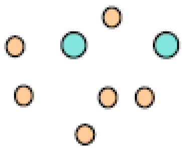
Start with the smaller number and count the gap to the larger number.

1 set within another (part-whole model)

The quantity in the whole set and one part are known, and may be used to find out how many are in the unknown part.

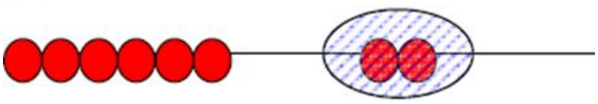
8 – 2 = 6

Counters:



Bead strings:

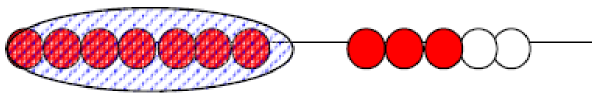
8 – 2 = 6



Bridging through 10s

This stage encourages children to become more efficient and begin employ known facts.

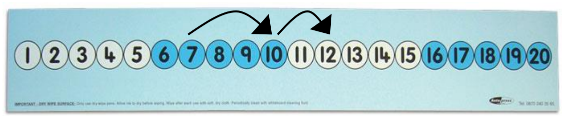
Bead string:



7 + 5 is decomposed/partitioned into 7 + 3 + 2.

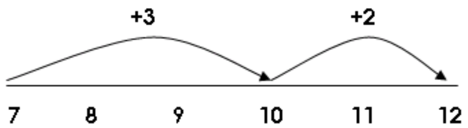
The bead sting illustrates 'how many more to the next multiple of 10?' (the children should identify how their number bonds are being applied) and then 'if we have used 3 of the 5 to get to 10, how many more do we need to add on? (ability to decompose/partition all numbers applied)

Number track:



Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number line

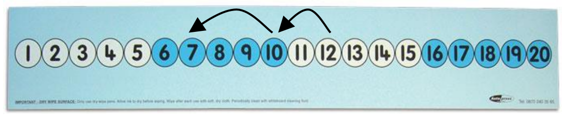


Bead string:

12 – 7 is decomposed/partitioned in 12 – 2 – 5.

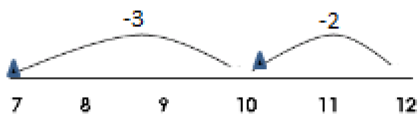
The bead string illustrates 'from 12 how many to the last/previous multiple of 10?' and then 'if we have used 2 of the 7 we need to subtract, how many more do we need to count back? (ability to decompose/partition all numbers applied)

Number Track:



Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number Line:



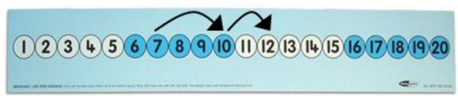
Counting up or 'Shop keepers' method

Bead string:

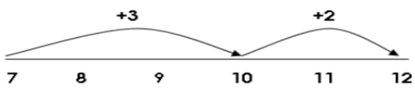
12 – 7 becomes 7 + 3 + 2.

Starting from 7 on the bead string 'how many more to the next multiple of 10?' (children should recognise how their number bonds are being applied), 'how many more to get to 12?'.

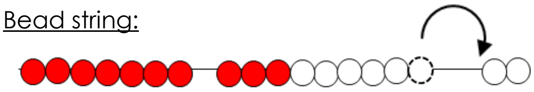
Number Track:



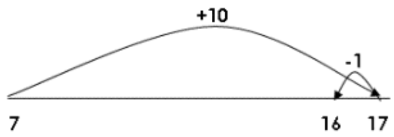
Number Line:



<table>
<tr><td colspan="2"><u>Compensation model (adding 9 and 11) (optional)</u>
This model of calculation encourages efficiency and application of known facts (how to add ten)</td></tr>
<tr><td>7 + 9

<u>Bead string:</u>


Children find 7, then add on 10 and then adjust by removing 1.

<u>Number line:</u>

</td>
<td>18 – 9

<u>Bead string:</u>


Children find 18, then subtract 10 and then adjust by removing 1.

<u>Number line:</u>

</td></tr>
</table>

Working with larger numbers

TO + TO

Ensure that the children have been transitioned onto Diennes and understand the abstract nature of the single 'tens' sticks and 'hundreds' blocks

Partitioning (Aggregation model)

34 + 23

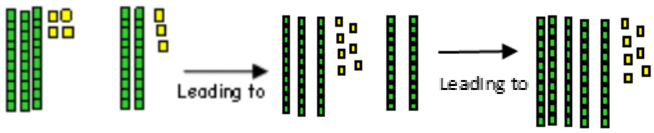
Diennes:

Children create the two sets with Diennes and then combine; ones with ones, tens with tens.

Partitioning (Augmentation model)

Diennes:

Encourage the children to begin counting from the first set of ones and tens, avoiding counting from 1. Beginning with the ones in preparation for formal columnar method.



Number line:

At this stage, children can begin to use an informal method to support, record and explain their method. (optional)

e.g.

34 + 23 =

30 4 20 3

30 + 20 = 50

4 + 3 = 7

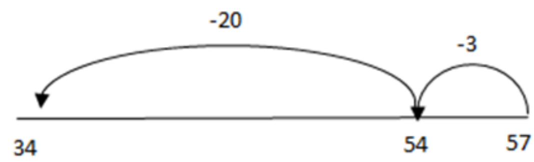
50 + 7 = 57

Take away (Separation model)

Children remove the lower quantity from the larger set, starting with the ones and then the tens. In preparation for formal decomposition.

57 – 23 = 34

Number Line:



At this stage, children can began to use an informal method to support, record and explain their method (optional)

e.g.

57 - 22 =

50 7 20 2

50 - 20 = 30

7 - 2 = 5

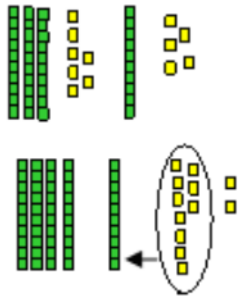
30 + 5 = 35

Bridging with larger numbers

Once secure in partitioning for addition, children begin to explore exchanging. What happens if the units are greater than 10? Introduce the term 'exchange'. Using the Diennes equipment, children exchange ten ones for a single tens rod, which is equivalent to crossing the tens boundary on the bead string or number line.

Diennes:

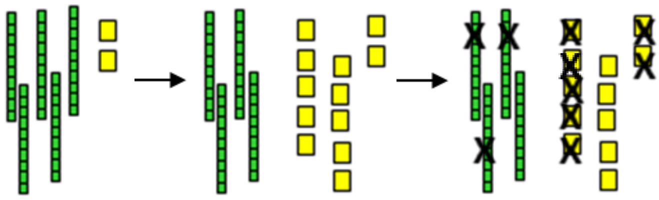
37 + 15



Discuss counting on from the larger number irrespective of the order of the calculation.

Diennes:

52 – 37 = 15

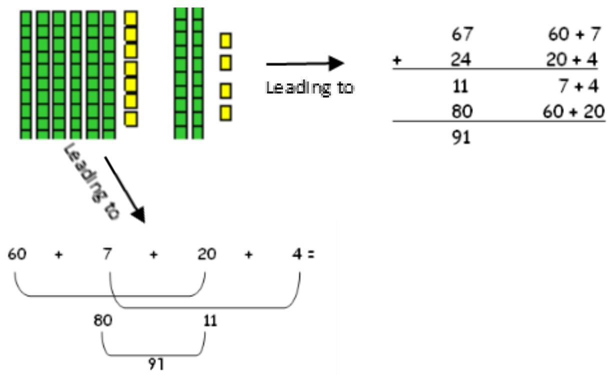


Expanded Vertical Method (optional)

Children are then introduced to the expanded vertical method to ensure that they make the link between using Diennes equipment, partitioning recording and the expanded vertical method.

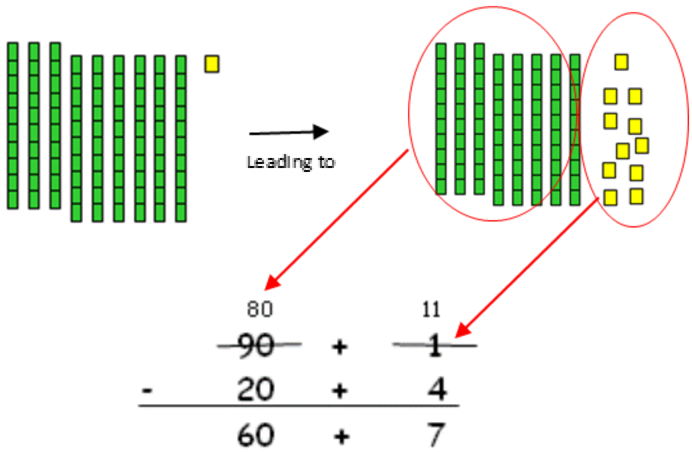
Diennes:

67 + 24 = 91

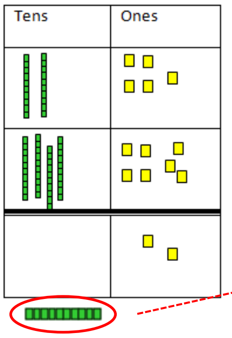


Diennes:

91 – 67 = 24

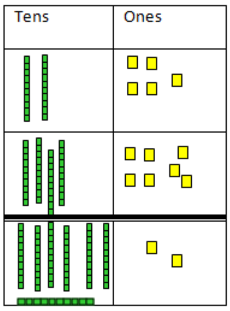


Compact method

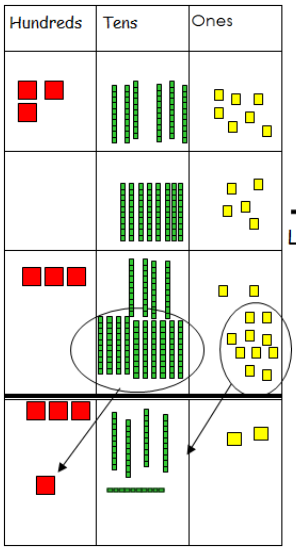


$$\begin{array}{r} 25 \\ 47 \\ \hline 2 \\ \hline {}_{1} \end{array}$$

Leading to



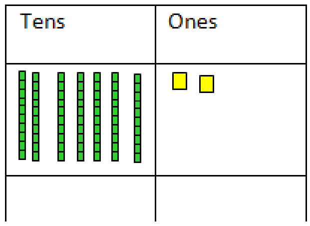
$$\begin{array}{r} 25 \\ 47 \\ \hline 72 \\ \hline {}_{1} \end{array}$$



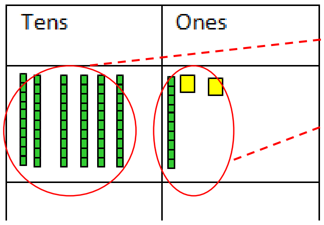
Leading to

$$\begin{array}{r} 3\ 6\ 7 \\ +\ 8\ 5 \\ \hline 4\ 5\ 2 \\ \hline {}_{1}\ {}_{1} \end{array}$$

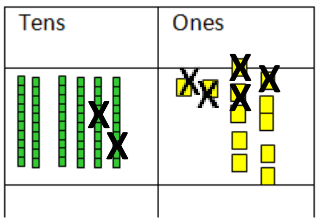
Compact decomposition



$$\begin{array}{r} 72 \\ -25 \\ \hline 47 \\ \hline \end{array}$$



$$\begin{array}{r} {}^{6}\cancel{7}{}^{1}2 \\ -25 \\ \hline \\ \hline \end{array}$$



$$\begin{array}{r} {}^{6}\cancel{7}{}^{1}2 \\ -25 \\ \hline 47 \\ \hline \end{array}$$

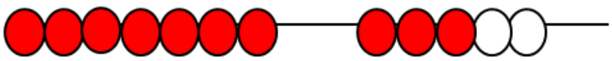
Vertical Acceleration

By returning to earlier manipulative experiences we support children in make links across mathematics, encouraging 'If I know this…then I also know…' thinking.

Decimals

Ensure that children are confident in counting forwards and backwards in decimals – using bead strings to support.

Bead strings:



Each bead represents 0.1, each different block of colour equal to 1.0

Diennes:

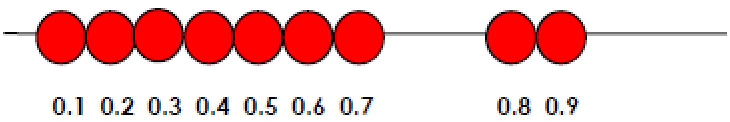
0.1 1.0 10.0

Addition of decimals

Aggregation model of addition

counting both sets– starting at zero.

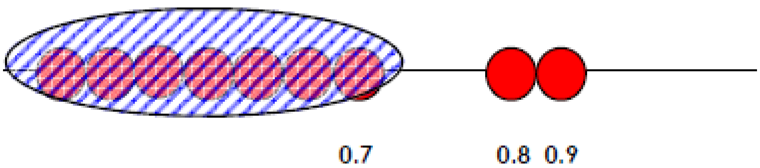
0.7 + 0.2 = 0.9



Augmentation model of addition:

starting from the first set total, count on to the end of the second set.

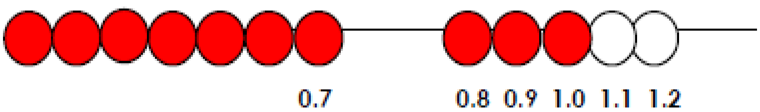
0.7 + 0.2 = 0.9



Bridging through 1.0

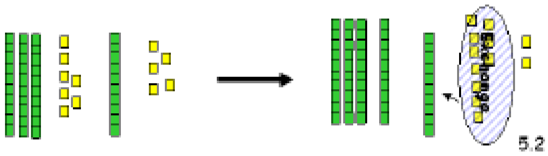
encouraging connections with number bonds.

0.7 + 0.5 = 1.2



Partitioning

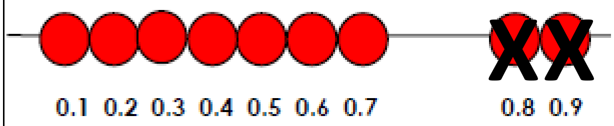
3.7 + 1.5 = 5.2



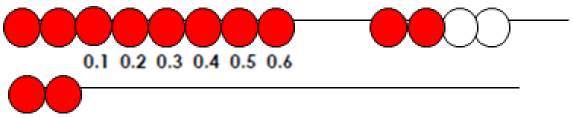
Subtraction of decimals

Take away model

0.9 – 0.2 = 0.7



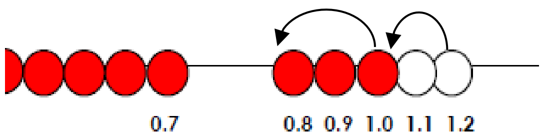
0.8 – 0.2 =



Bridging through 1.0

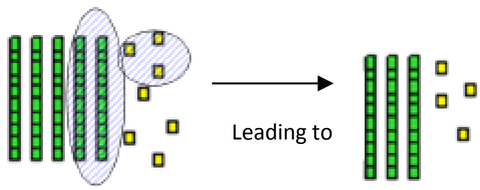
encourage efficient partitioning.

1.2 – 0.5 = 1.2 – 0.2 – 0.3 = 0.7



Partitioning

5.7 – 2.3 = 3.4



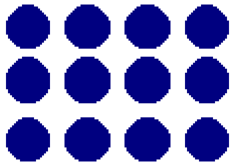
Gradation of difficulty- addition	**Gradation of difficulty- subtraction**
1. No exchange 2. Extra digit in the answer 3. Exchanging ones to tens 4. Exchanging tens to hundreds 5. Exchanging ones to tens and tens to hundreds 6. More than two numbers in calculation 7. As 6 but with different number of digits 8. Decimals up to 2 decimal places (same number of decimal places) 9. Add two or more decimals with a range of decimal places.	1. No exchange 2. Fewer digits in the answer 3. Exchanging tens for ones 4. Exchanging hundreds for tens 5. Exchanging hundreds to tens and tens to ones 6. As 6 but with different number of digits 7. Decimals up to 2 decimal places (same number of decimal places) 8. Subtract two or more decimals with a range of decimal places.

Progression in Multiplication and Division

Multiplication and division are connected.

Both express the relationship between a number of equal parts and the whole.

Part	Part	Part	Part
Whole			



The following array, consisting of four columns and three rows, could be used to represent the number sentences: -

3 x 4 = 12,

4 x 3 =12,

3 + 3 + 3 + 3 = 12,

4 + 4 + 4 =12.

And it is also a model for division

12÷4 = 3

12÷3 = 4

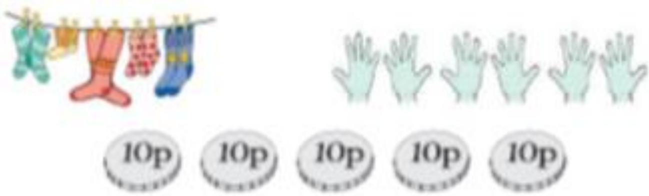
12 – 4 – 4 – 4 = 0

12 – 3 – 3 – 3 – 3 = 0

MULTIPLICATION	DIVISION

Early experiences

Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups.

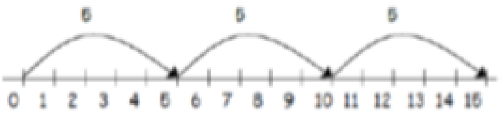


Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s.

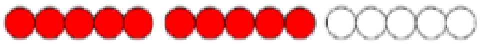
Repeated addition (repeated aggregation)

3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5 x 3

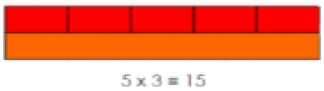
Children learn that repeated addition can be shown on a number line.



Children learn that repeated addition can be shown on a bead string.

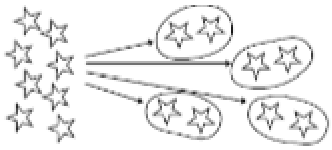


Children also learn to partition totals into equal trains using Cuisenaire Rods



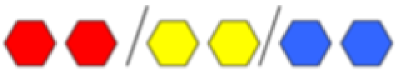
Sharing equally

6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses.



Grouping or repeated subtraction

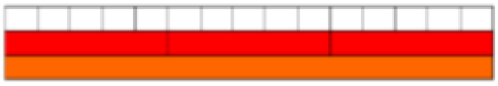
There are 6 sweets. How many people can have 2 sweets each?



Scaling

This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles.

This can be written as:
1 + 1 + 1 = 3 →scaled up by 3 →5 + 5 + 5 = 15

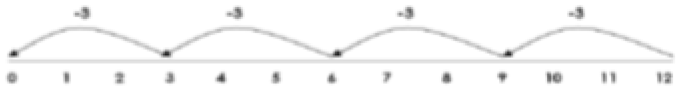


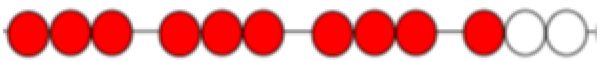
For example, find a ribbon that is 4 times as long as the blue ribbon.

We should also be aware that if we multiply by a number less than 1, this would correspond to a scaling that reduces the size of the quantity. For example, scaling 3 by a factor of 0.5 would reduce it to 1.5, corresponding to 3 X 0.5 = 1.5.

Repeated subtraction using a bead string or number line

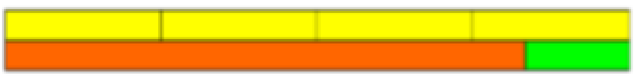
12 ÷ 3 = 4





The bead string helps children with interpreting division calculations, recognising that 12 ÷ 3 can be seen as 'how many 3s make 12?'

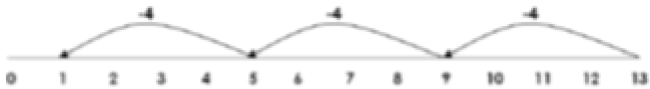
Cuisenaire Rods also help children to interpret division calculations.



Grouping involving remainders

Children move onto calculations involving remainders.

13 ÷ 4 = 3 r1



Or using a bead string see above.

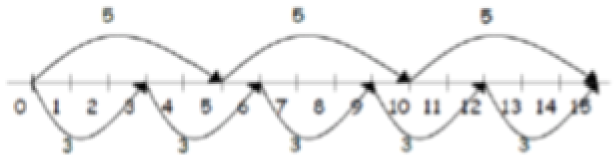
Commutativity

Children learn that 3 x 5 has the same total as 5 x 3.

This can also be shown on the number line.

3 x 5 = 15

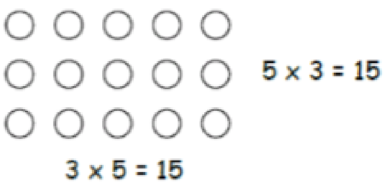
5 x 3 = 15



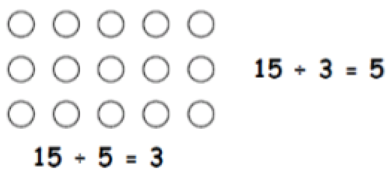
Children learn that division is **not** commutative and link this to subtraction.

Arrays

Children learn to model a multiplication calculation using an array. This model supports their understanding of **commutativity** and the development of the grid in a written method. It also supports the finding of factors of a number.



Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' in a written method. This model also connects division to **finding fractions** of discrete quantities.



Inverse operations

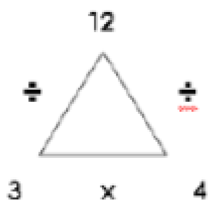
'Magic triangles' can be used to understand factor families and to model the 4 related multiplication and division facts. Children learn to state the 4 related facts.

3 x 4 = 12

4 x 3 = 12

12 ÷ 3 = 4

12 ÷ 4 = 3

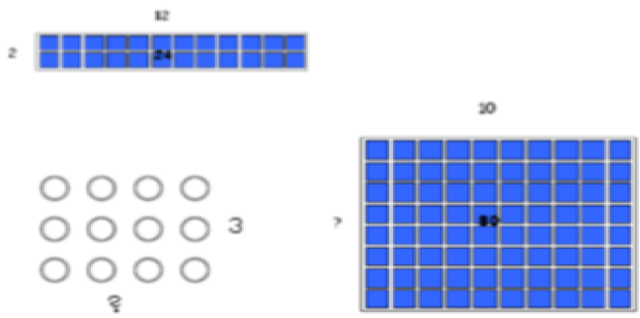


Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

□ x 5 = 20   3 x Δ = 18   O x □ = 32

24 ÷ 2 = □   15 ÷ O = 3   Δ ÷ 10 = 8

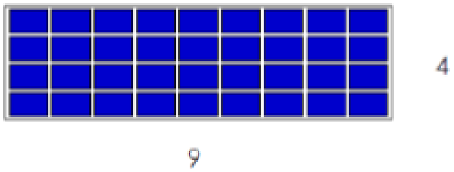
This can also be supported using arrays: e.g. 3 X ? = 12



Partitioning for multiplication

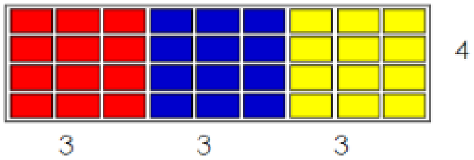
Arrays are also useful to help children visualise how to partition larger numbers into more useful arrays.

9 x 4 = 36



Children should be encouraged to be flexible with how they use number and can be encouraged to break the array into more manageable chunks.

9 x 4 =

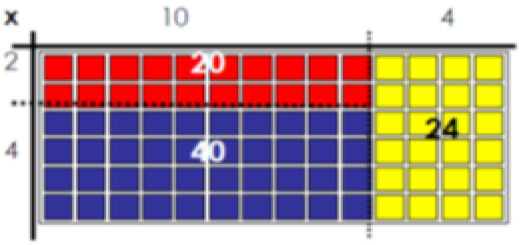


Which could also be seen as

9x 4 = (3 x 4) + (3 x 4) + (3 x 4) = 12 + 12 + 12 = 36

Or 3 x (3x4) = 36

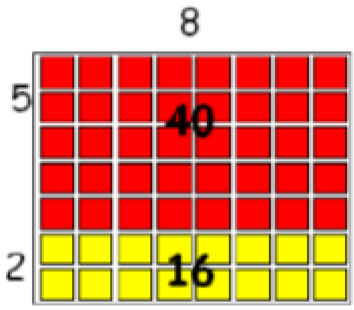
And so 6 x 14 = (2 x 10) + (4 x 10) + (4 x 6) = 20 + 40 + 24 = 84



Partitioning for division

The array is also a flexible model for division of larger numbers

56 ÷ 8 = 7



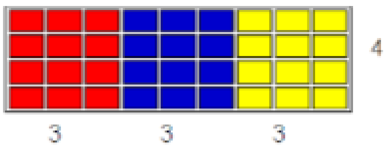
Children could break this down into more manageable arrays, as well as using their understanding of the inverse relationship between division and multiplication.

56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8) = 5 + 2 = 7

To be successful in calculation learners must have plenty of experiences of being flexible with partitioning, as this is the basis of distributive and associative law.

Associative law (multiplication only) :-

E.g. 3 x (3x4) = 36

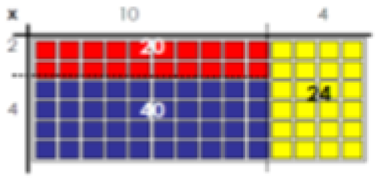


The principle that if there are three numbers to multiply these can be multiplied in any order.

Distributive law (multiplication):-

E.g. 6 x 14 = (2 x 10) + (4 x 10) + (4 x 6) = 20 + 40 + 24 = 84

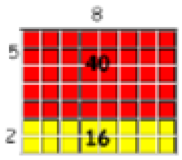
This law allows you to distribute a multiplication across an addition or subtraction.



Distributive law (division):-

E.g. 56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8) = 5 + 2 = 7

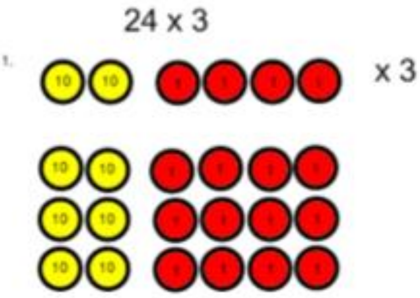
This law allows you to distribute a division across an addition or subtraction.



Arrays leading into the grid method

Children continue to use arrays and partitioning where appropriate, to prepare them for the grid method of multiplication.

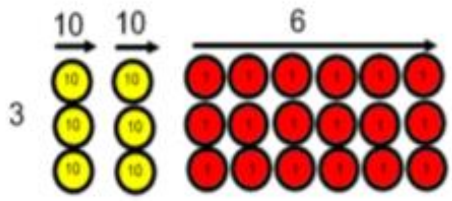
Arrays can be represented as ‘grids’ in a shorthand version and by using place value counters we can show multiples of ten, hundred etc.



Arrays leading into chunking and then long and short division

Children continue to use arrays and partitioning where appropriate, to prepare them for the ‘chunking’ and short method of division. Arrays are represented as ‘grids’ as a shorthand version.

e.g. 78 ÷ 3 =

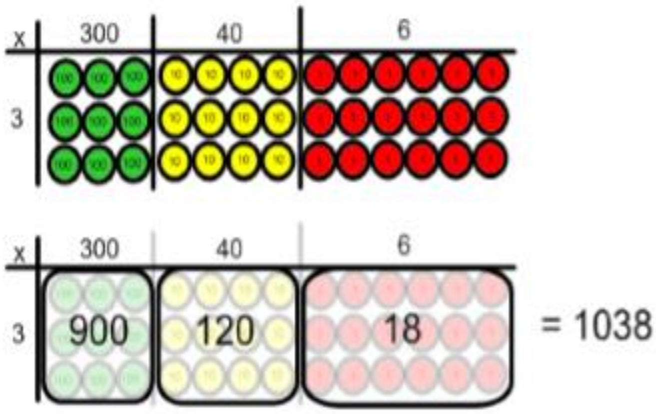


78 ÷ 3 = (30 ÷ 3) + (30 ÷ 3) + (18 ÷ 3) =

10 + 10 + 6 = 26

Grid method

This written strategy is introduced for the multiplication of TU x U to begin with. It may require column addition methods to calculate the total.



The vertical method- ‘chunking’ leading to long division

Some children may find it useful to learn this method, particularly if modelled as an array using place value counters (see above).

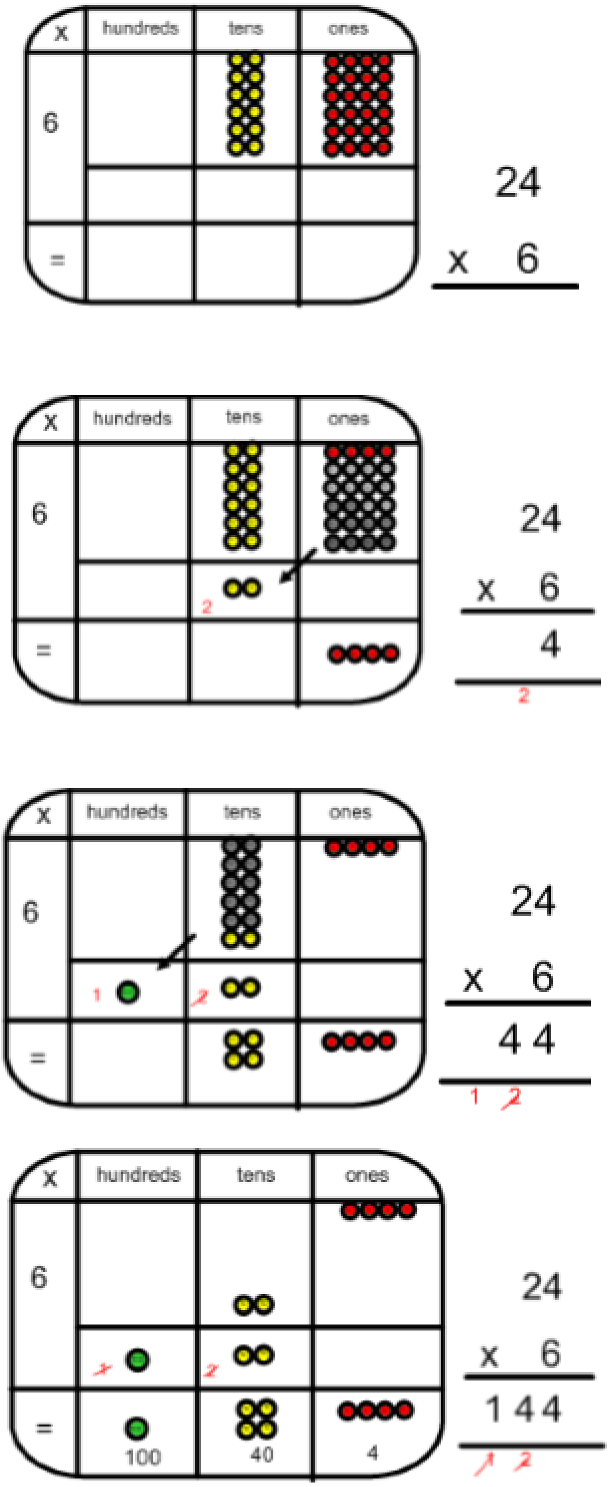
78 ÷ 3 =

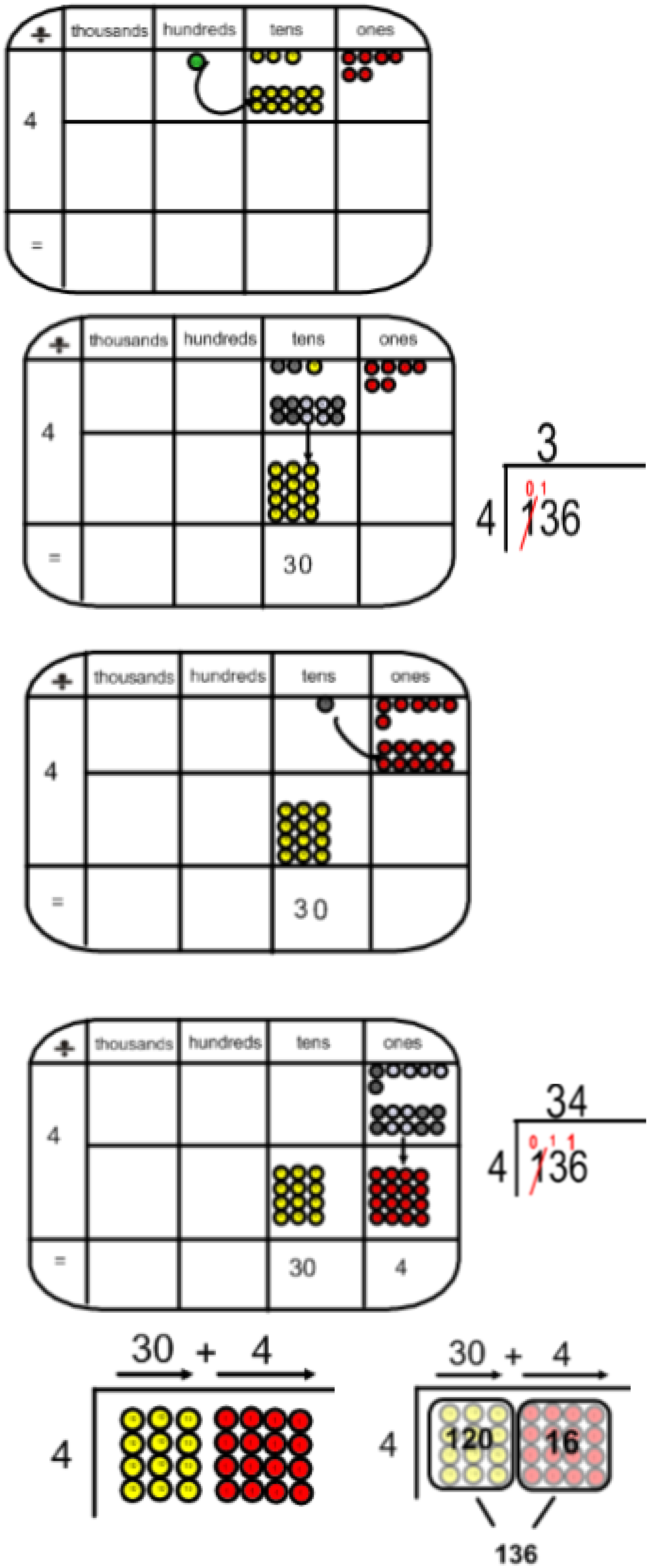
	78	
-	30	(10 x 3)
	48	
-	30	(10 x 3)
	18	
-	18	(6 x 3)
	0	

So 78 ÷ 3 = 10 + 10 + 6 =26

[

Short multiplication—multiplying by a single digit	**Short division —dividing by a single digit**
The array using place value counters becomes the basis for understanding short multiplication first without exchange before moving onto exchanging	Whereas we can begin to group counters into an array to show short division working
24 x 6	136 ÷ 4





Gradation of difficulty (Short multiplication)

1. TO x O no exchange
2. TO x O extra digit in the answer
3. TO x O with exchange of ones into tens
4. HTO x O no exchange
5. HTO x O with exchange of ones into tens
6. HTO x O with exchange of tens into hundreds
7. HTO x O with exchange of ones into tens and tens into hundreds
8. As 4-7 but with greater number digits x O
9. O.t x O no exchange
10. O.t with exchange of tenths to ones
11. As 9 - 10 but with greater number of digits which may include a range of decimal places x O

Gradation of difficulty (Short division)

1. TO ÷ O no exchange no remainder
2. TO ÷ O no exchange with remainder
3. TO ÷ O with exchange no remainder
4. TO ÷ O with exchange, with remainder
5. Zeroes in the quotient e.g. 816 ÷ 4 = **204**
6. As 1-5 HTO ÷ O
7. As 1-5 greater number of digits ÷ O
8. As 1-5 with a decimal dividend e.g. 7.5 ÷ 5 or 0.12 ÷ 3
9. Where the divisor is a two digit number

See below for gradation of difficulty with remainders

Dealing with remainders

Remainders should be given as integers, but children need to be able to decide what to do after division, such as rounding up or down accordingly.

e.g.

· I have 62p. How many 8p sweets can I buy?

· Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed?

Gradation of difficulty for expressing remainders

1. Whole number remainder
2. Remainder expressed as a fraction of the divisor
3. Remainder expressed as a simplified fraction
4. Remainder expressed as a decimal

Multiplying by more than one digit

Children may refer back to the grid method and visuals as above and compare before being required to record as:

$$\begin{array}{r} 34 \\ \times\ 12 \\ \hline 340 \\ 68 \\ \hline 368 \end{array}$$

(Always start with the tens)